Penurunan salah satu Rumus Transformasi Laplce

Buktikan : $L\{t^n \cdot F(t)\} = (-1)^n \cdot f^{(n)}(s)$

Bukti :

$$L\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dt = f(s)$$

atau $f(s) = \int_0^\infty e^{-st} \cdot F(t)\, dt$

$f(s)$ **didiferential terhadap fungsi s.**

$$f^{(1)}(s) = \int_0^\infty e^{-st} (-t) \cdot F(t)\, dt$$

$$f^{(1)}(s) = -\int_0^\infty e^{-st} \{t \cdot F(t)\}\, dt \quad \text{...(1)}$$

$$f^{(1)}(s) = (-1)\, L\{t \cdot F(t)\}$$

$$L\{t \cdot F(t)\} = \frac{f^{(1)}(s)}{(-1)}$$

$$\boxed{L\{t \cdot F(t)\} = (-1)^1 \cdot f^{(1)}(s)}$$

(1) → $f^{(1)}(s) = -\int_0^\infty e^{-st} \{t \cdot F(t)\}\, dt$

$$f^{(2)}(s) = -\int_0^\infty e^{-st} (-t) \{t \cdot F(t)\}\, dt$$

$$f^{(2)}(s) = \int_0^\infty e^{-st} \{t^2 \cdot F(t)\}\, dt \quad \text{...(2)}$$

$$f^{(2)}(s) = L\{t^2 \cdot F(t)\}$$

atau $L\{t^2 \cdot F(t)\} = f^{(2)}(s)$

$$\boxed{L\{t^2 \cdot F(t)\} = (-1)^2 \cdot f^{(2)}(s)}$$

(2) → $f^{(2)}(s) = \int_0^{\infty} e^{-st}\{t^2 \cdot F(t)\}dt$

$$f^{(3)}(s) = \int_0^{\infty} e^{-st}(-t)\{t^2 \cdot F(t)\}dt$$

$$f^{(3)}(s) = -\int_0^{\infty} e^{-st}\{t^3 \cdot F(t)\}dt$$

$$f^{(3)}(s) = -L\{t^3 \cdot F(t)\}$$

$$L\{t^3 \cdot F(t)\} = -f^{(3)}(s)$$

$$\boxed{L\{t^3 \cdot F(t)\} = (-1)^3 \cdot f^{(3)}(s)}$$

Kesimpulan :

$$L\{t \cdot F(t)\} = (-1)^1 \cdot f^{(1)}(s)$$

$$L\{t^2 \cdot F(t)\} = (-1)^2 \cdot f^{(2)}(s)$$

$$L\{t^3 \cdot F(t)\} = (-1)^3 \cdot f^{(3)}(s)$$

Analog :

$$\boxed{L\{t^n \cdot F(t)\} = (-1)^n \cdot f^{(n)}(s)}$$

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